

Name:

Exam 1 – 9/27/2023

Instructions

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
2	1	
3a	1	
3b	1	
3c	1	
3d	1	
4	2	
Total		/ 100

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Signature:

Problem 1. Movr is a ride sharing service that operates in Simplexville. They have hired you to study the movement of its cars between three regions in Simplexville: Uptown, Midtown, and Downtown. The company's data analytics team has modeled the movement of a car as a Markov chain with 3 states. The states 1, 2, 3 correspond to Uptown, Midtown, and Downtown, respectively, and each time step corresponds to one car trip. When a car reaches its destination, it stays in the destination region until it is used again. The one-step transition probability matrix for a car is:

$$\mathbf{P} = \begin{bmatrix} 0.35 & 0.47 & 0.18 \\ 0.18 & 0.63 & 0.19 \\ 0.19 & 0.52 & 0.29 \end{bmatrix}$$

Suppose at the beginning of the day, 30% of the cars are in the Uptown region, 45% in the Midtown region, and 25% in the Downtown region.

- a. What is the probability that a car randomly chosen at the beginning of the day will be in the Midtown region after 6 trips? Provide your answer to 3 decimal places.

- b. What is the probability that a car starting in the Downtown region will be in the Uptown region after 2 trips? Provide your answer to 3 decimal places.

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Here is the transition probability matrix for Problem 1 again:

$$\mathbf{P} = \begin{bmatrix} 0.35 & 0.47 & 0.18 \\ 0.18 & 0.63 & 0.19 \\ 0.19 & 0.52 & 0.29 \end{bmatrix}$$

- c. What is the probability that a car starts in the Uptown region, stays in either the Uptown or Downtown regions for 3 trips, and then goes to the Midtown region in the 4th trip? Provide your answer to 3 decimal places.

Problem 2. You have just been hired as an analyst in the Cauchy County Department of Health and Human Services. Your predecessor developed a model of the county population, in which each citizen can be classified as living in one of three location types: urban, rural, or suburban. In their model, the state of the system is defined as a citizen's current location type, and the time step is defined to be 1 year.

Describe what assumptions need to be made in order for the Markov property to hold. (You do not need to discuss whether these assumptions are realistic.)

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Here is the transition probability matrix for Problem 3 again:

$$\mathbf{P} = \begin{bmatrix} 0.10 & 0.25 & 0.15 & 0.10 & 0.05 & 0.35 \\ 0 & 0.25 & 0.65 & 0 & 0 & 0.10 \\ 0 & 0.65 & 0.20 & 0 & 0 & 0.15 \\ 0.20 & 0.10 & 0.40 & 0.10 & 0.15 & 0.05 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.20 & 0.10 & 0 & 0 & 0.70 \end{bmatrix}$$

c. Suppose the UUV starts in region 4. What is the probability that the UUV eventually ends up in region 5? Provide your answer to 3 decimal places.

d. What is the expected time to absorption from state 4? Provide your answer to 3 decimal places.

Problem 4. Arrow Advertising wants to study how consumers behave in its direct marketing campaign for Primal Pralines. According to historical data, the probability that a customer will purchase a box of pralines in response to a “remarketing” phone call depends on the number of months have passed since the customer last purchased a box.

If one month has passed since the customer’s last purchase, the customer will purchase another box 70% of the time. If two months have passed, the customer will purchase another box 50% of the time. If three months have passed, the customer will purchase another box 30% of the time. Finally, once four months have passed, the firm will stop all future efforts to “remarket” to the customer.

One month has passed since last purchase for 40% of the customers involved. Two months have passed for 30%, and three months have passed for 20%. Finally, four months or longer have passed for 10% of the customers involved.

A customer purchases at most one box of pralines per month.

Model this setting as a Markov chain by defining:

- the state space and the meaning of each state,
- the meaning of one time step in the setting’s context,
- the one-step transition probabilities, and
- the initial state probabilities.

Specify the one-step transition probabilities as a matrix, and the initial state probabilities as a vector.